DETERMINATION OF THE EFFECTIVE PERMEABILITY OF A CRACKED-POROUS MEDIUM

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Formulas are derived to calculate the effective permeability of periodic, highly nonuniform porous structures. Also, methods are developed to calculate the parameters of the medium from data from studies of wells.

A cracked-porous medium is taken to mean a porous medium with a developed system of interconnected cracks. The filtration of a slightly compressible fluid in a cracked-porous medium is described either within the framework of the model of two interpenetrating continua [1, 2] or by a model of filtration in a system of cracks in which the blocks of material containing the cracks are represented as distributed sources [3-5]. It follows from the results of these studies that there are three characteristic stages in the filtration of a slightly compressible fluid. In the first stage, motion takes place in a system of cracks; in the second stage, there is an exchange of fluid between the blocks and the cracks; in the third stage, the cracked-porous medium behaves as though it were uniform. In studying a well at the first stage, a determination is made of the permeability of the system of connected cracks (the effective permeability of a medium with an impermeable matrix), while the effective permeability of a cracked-porous medium is determined in studies of the third stage. It is interesting to find the relationship between these effective permeabilities and the parameters of the internal structure of the reservoir — the permeability of the material of the blocks and cracks, the mean lengths of the blocks, and the openness of the cracks.

The effective permeability of a purely crack-like medium (with impermeable blocks) was determined in [4]. The authors of [3, 5] calculated the effective permeability of a multilayered bed periodically composed of rocks of different permeabilities. In [5], the model of a cracked-porous medium with blocks in the form of distributed sources was used to obtain a formula to determine effective permeability.

It should be noted that it was assumed in the models in [1, 2] and [3-5] that filtration through a system of cracks occurs in accordance with Darcy's law and that the permeability of the cracks is independent of the permeability of the blocks.

Here, we examine filtration in a cracked-porous medium within the framework of the model of filtration in a porous medium with periodic, rapidly-oscillating properties [6, 7]. It is shown that the permeability coefficient in the formula for fluid flow in the system of cracks depends not only on the permeability of the cracked material, but also on the permeability of the blocks. It is established that there is interaction between the filtration flows in the blocks and the cracks ("suppression" effect). Formulas are obtained expressing the effective permeabilities of a cracked porous medium and a purely cracklike system through the parameters of the internal structure. Using these formulas and data from studies of wells, we then determine the geometric and filtration parameters of the internal structure of a cracked-porous medium.

1. Formulation of the Problem. The motion of a homogeneous, slightly compressible fluid in a cracked-porous medium is described by the equation of nonsteady filtration in a porous medium [1] with rapidly oscillating coefficients [7] (this reflects the sharp difference in the properties of the medium in the blocks and the cracks):

$$m\left(\frac{x}{\varepsilon}\right)\frac{\partial p}{\partial t}=\frac{1}{\mu\beta}\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\left(k\left(\frac{x}{\varepsilon}\right)\frac{\partial p}{\partial x_i}\right),\ \varepsilon=(\Delta+l)/L\ll 1. \tag{1}$$

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Here, $k(x_1, x_2, x_3)$, $m(x_1, x_2, x_3)$ are piecewise-constant functions that are periodic with respect to the three variables. The period is ε.

Equation (1) is satisfied everywhere outside the surface of discontinuity of the functions $k(x)$ and $m(x)$. On the surface, the conditions of continuity of pressure and the volume flow of the fluid are satisfied:

$$[p]=0; \left[k\frac{\partial p}{\partial n}\right]=0. \tag{2}$$

The filtration of a slightly compressible homogeneous fluid is described by the following mixed problem for Eq. (1): the initial condition $p(x) = p_*$ corresponds to the undisturbed state of the bed before exploitation. Conditions of impermeability are imposed at the boundary of the filtration region (bed), while first-order (pressure $p_w/(t)$) or second-order (volume flow rate of fluid $q(t)$) conditions are imposed on the internal contours (wells).

To study filtration at the third stage, after the completion of exchange between the blocks and cracks, we will use the method of averaging processes in periodic media [7]. Together with the slow variables x_i, we introduce fast variables $\xi_i = x_i/\varepsilon$, $\xi_i \in [0, 1]$. We seek an asymptotic solution of problem (1) $p(x, \xi)$, periodic with respect to the variables ξ_i, in the form of a series in powers of the small parameter ε:

$$p(x, \xi)=p_0(x)+\sum_{i=1}^{3} N_i(\xi)\frac{\partial p_0}{\partial x_i}+\sum_{i=2}^{\infty}\varepsilon^i p_i(x, \xi). \tag{3}$$

To within terms on the order of $O(\sqrt{\varepsilon})$, the solution of problem (1-2) coincides with the smooth function $p_0(x_1, x_2, x_3)$ (3), which is not dependent of the fast variables ξ_i and which satisfies the solution of the "averaged" problem with constant coefficients $\hat{m}$ and $\hat{K}_{ij}$:

$$\hat{m}\frac{\partial p_0}{\partial t}=\frac{1}{\mu\beta}\sum_{i,j=1}^{3}\hat{K}_{ij}\frac{\partial^2 p_0}{\partial x_i \partial x_j}. \tag{4}$$

The initial and boundary conditions for the function $p_0(x)$ are the same as for the function $p(x)$.

Periodic functions $N_i(\xi)$ (3) are determined from the solution of the following boundary-value problem, referred to as the "cell problem:"

$$\sum_{j=1}^{3}\frac{\partial}{\partial \xi_j}\left(k(\xi)\frac{\partial}{\partial \xi_j}(N_i+\xi_i)\right)=0,\ 0\leqslant \xi_i \leqslant 1, \tag{5}$$

with conditions of continuity of pressure and flow rate on the surfaces of discontinuity

$$[N_i]=0, \left[\sum_{j=1}^{3} k(\xi)\frac{\partial}{\partial \xi_j}(N_i+\xi_i)n_j\right]=0. \tag{6}$$

For the sake of determinateness, we assume that $N_i = 0$ on the boundary of the cell ∂I^3.

The values of the effective coefficients $\hat{K}_{ij}$ are expressed through the solution of the cell problem in the following manner:

$$\hat{K}_{ij}=\int_{I^3} k(\xi)\frac{\partial}{\partial \xi_i}(N_j+\xi_j)\,d\xi, \tag{7}$$

$$\hat{m}=\int_{I^3} m(\xi)\,d\xi. \tag{8}$$

The total fluid flow W satisfies the relation:

$$\mathbf{W}_i = -\frac{1}{\mu}\sum_{j=1}^{3}\hat{K}_{ij}\frac{\partial p}{\partial x_j}.$$

The tensor $\hat{K}_{ij}$ is called the tensor of the effective permeability of the medium.

It should be noted that the accuracy of calculation of the filtration flow from the first two terms of asymptotic expansion (3) is equal to $O(\sqrt{\varepsilon})$ [7].

The fluid flow W through a unit area of the medium is composed of the flow through the system of cracks $W^{(1)}$ and the flow through the matrix $W^{(2)}$:

$$\mathbf{W} = \mathbf{W}^{(1)} + \mathbf{W}^{(2)}. \tag{9}$$

We will calculate these flows with the accuracy $O(\sqrt{\varepsilon})$. To do this, we integrate the volume flow of the fluid by parts of the cell belonging to blocks Γ_2 and cracks Γ_1:

$$\mathbf{W}_i^{(r)} = -\frac{1}{\mu}\int_{\Gamma_r} k(\xi)\frac{\partial p}{\partial x_i}d\xi = -\frac{k_r}{\mu}\int_{\Gamma_r}\left(\frac{\partial p_0}{\partial x_i} + \right.$$
$$\left. + \sum_{j=1}^{3}\frac{\partial N_j}{\partial \xi_i}\frac{\partial p_0}{\partial x_j}\right)d\xi = -\frac{k_r}{\mu}\left\{\int_{\Gamma_r}\left(1+\frac{\partial N_i}{\partial \xi_i}\right)d\xi\,\frac{\partial p_0}{\partial x_i} + \sum_{i\neq j}\int_{\Gamma_r}\frac{\partial N_j}{\partial \xi_i}d\xi\,\frac{\partial p_0}{\partial x_j}\right\}. \tag{10}$$

Thus, after averaging of the flow in accordance with the "isotropic" Darcy's law (1), we obtain an anisotropic flow both within the system as a whole (7) and in each phase (10):

$$\mathbf{W}_i^{(r)} = -\sum_{j=1}^{3}\frac{\hat{K}_{ij}^{(r)}}{\mu}\frac{\partial p_0}{\partial x_j};\ \hat{K}_{ij}^{(r)} = k_r\int_{\Gamma_r}\frac{\partial}{\partial \xi_i}(N_j+\xi_j)\,d\xi. \tag{11}$$

It is evident from (9-11) that $\hat{K}_{ij} = \hat{K}_{ij}^{(1)} + \hat{K}_{ij}^{(2)}$. The tensor $\hat{K}_{ij}$ is referred to as the crack permeability tensor, while $\hat{K}_{ij}^{(2)}$ is referred to as the tensor of the effective permeability of the matrix. By virtue of boundary-value problem (5-6), the components of the tensors $\hat{K}_{ij}$ and $\hat{K}_{ij}^{(r)}$ are functions of the intrinsic permeabilities of the cracks k_1 and blocks k_2.

It is evident from Eqs. (7) and (11) that the following relations are satisfied for a purely cracklike system (with impermeable blocks, $k_2 = 0$)

$$\hat{K}_{ij}^{(2)}(k_1,\ 0) = 0,\ \hat{K}_{ij}(k_1,\ 0) = \hat{K}_{ij}^{(1)}(k_1,\ 0). \tag{12}$$

Let us calculate the permeabilities of cracked-porous media with different internal structures.

2. Effective Permeability of a Multilayered Bed. We will examine the filtration of a homogeneous elastic fluid in a layered-nonuniform bed. Let the rock above the bed be located along the axis x_3. We will designate the other axes in the horizontal plane as x_1 and x_2.

The dynamics of the pressure distribution is described by Eq. (1), where the permeability function is a rapidly oscillating piecewise-constant function of ξ_3. In this case, $N_1 = N_2 = 0$.

Boundary-value problem (5)-(6) takes the following form for N_3

$$\begin{gathered}\frac{\partial}{\partial \xi_3}\left(k(\xi_3)\frac{\partial}{\partial \xi_3}(N_3+\xi_3)\right) = 0,\\ N_3(0) = N_3(1) = 0,\ N_3(\xi_3)|_{\delta-0} = N_3(\xi_3)|_{\delta+0},\\ k_2\frac{\partial}{\partial \xi_3}(N_3(\xi_3)+\xi_3)|_{\delta-0} = k_1\frac{\partial}{\partial \xi_3}(N_3(\xi_3)+\xi_3)|_{\delta+0}.\end{gathered} \tag{13}$$

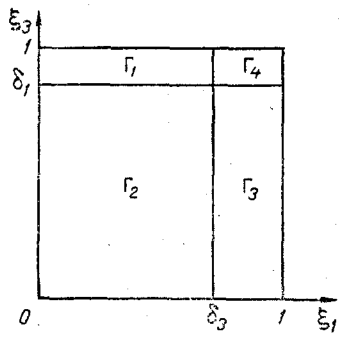

Fig. 1. Formulation of the cell problem for a layered-inhomogeneous bed dissected by a family of vertical cracks.

The solution of boundary-value problem (13) will be as follows:

$$N_3(\xi_3) = \begin{cases} (1-\delta)(k_1-k_2)\,\xi_3/[\delta k_1 + (1-\delta)\,k_2], & 0<\xi_3<\delta, \\ \delta\,(k_1-k_2)(1-\xi_3)/[\delta k_1+(1-\delta)\,k_2], & \delta<\xi_3<1. \end{cases} \tag{14}$$

Proceeding on the basis of (14), we use Eqs. (7) to find the coefficients of the tensor:

$$\hat{K}_{11} = \hat{K}_{22} = k_1(1-\delta) + k_2\delta,\ \hat{K}_{ij} = 0,\ i \neq j,$$
$$\hat{K}_{33} = k_1k_2/[\delta k_1 + (1-\delta)\,k_2]. \tag{15}$$

Thus, the permeability tensor of a multilayered bed, calculated with the accuracy $0(\sqrt{\varepsilon})$, is diagonal. The permeabilities in two horizontal directions are equal to the arithmetic mean permeabilities of the partings. With a small opening of the cracks $\Delta \ll \ell$ and a crack permeability k_1 as large as desired, the permeability in the vertical direction $\hat{K}_{33}$ approaches the value k_2.

With an impermeable matrix, there is no filtration in the vertical direction in a purely cracklike system, $\hat{K}_{33}(k_1, 0) = 0$. Here, $\hat{K}_{11}(k_1, 0) = \hat{K}_{22}(k_1, 0) = k_1(1-\delta)$. This agrees with the result in [4].

Let us determine the effective permeability of a multilayered bed dissected by a system of parallel vertical cracks. Of the horizontal axes, x_1 lies in the plane of the vertical cracks and x_2 is perpendicular to this plane. Further, k_2 is the permeability of the matrix and δ_1 and δ_3 are respectively the height and width of a block; k_1 is the permeability of the horizontal cracks and k_3 is the permeability of the vertical cracks.

In this case, $N_2 = 0$. Boundary-value problems (5-6) for $i = 1, 3$ are formulated on the plane (ξ_1, ξ_3) (see Fig. 1). We will examine the case when the openness of the cracks is much less than the dimensions of the blocks in the corresponding directions: $\Delta_1 \gg l_1$, $\Delta_3 \ll l_3$. We will prove the theorem that the asymptote of the solution of problem (5-6) in the small parameters ε, Δ_1/l_1 and Δ_3/l_3 has the following form:

$$N_1(\xi_1) = \begin{cases} \alpha_1\xi_1\rho_1(\xi_1), & \xi \in \Gamma_1 \cup \Gamma_4, \\ \alpha_1\xi_1, & \xi \in \Gamma_2, \\ \alpha_2(1-\xi_1)\,\rho_3(\xi_3), & \xi \in \Gamma_3. \end{cases} \tag{16}$$

Here, $\rho_i(\xi_i)$ is the characteristic function of the set $[0, \delta_i] \subset [0, 1]$ smoothed in a small neighborhood of the point $\delta_i \in [0, 1]$; $i = 1, 3$. Thus, $N_1 = 0$ when $\xi \in \Gamma_4$. The function N_1 depends on the variable ξ_1.

The coefficients α_1 and α_2 are equal to:

$$\alpha_1 = \frac{(1-\delta_3)(k_3-k_2)}{\delta_3 k_3 + (1-\delta_3)k_2}, \quad \alpha_2 = \frac{\delta_3(k_3-k_2)}{\delta_2 k_3 + (1-\delta_3)k_2}. \tag{17}$$

By replacing the subscript $i = 1$ by $i = 3$ in Eqs. (15-17), we obtain an expression for the function $N_3(\xi_3)$. This theorem was proved in the case $k_2 = 0$ in [7]. Now we use Eq. (7) to find the elements of the tensor of effective permeability of the cracked-porous medium. Calculated with the accuracy $O((1-\delta_1)(1-\delta_3))$ attained in the present study, the tensor has nondiagonal elements equal to zero. For the diagonal elements, we obtain the following expressions:

$$\hat{K}_{11} = \frac{k_3[k_1(1-\delta_1)+k_2\delta_1]}{\delta_3 k_3 + (1-\delta_3)k_2}, \quad \hat{K}_{33} = \frac{k_1[k_3(1-\delta_3)+k_2\delta_3]}{\delta_1 k_1 + (1-\delta_1)k_2},$$

$$\hat{K}_{22} = k_1(1-\delta_1)\delta_3 + k_2\delta_1\delta_3 + k_3(1-\delta_3).$$

With the same accuracy, the tensors of crack permeability and the effective permeability of the matrix are diagonal. For the system of cracks, we have

$$\hat{K}_{11}^{(1)} = \frac{k_3[k_2\delta_1(1-\delta_3)+k_1(1-\delta_1)]}{\delta_3 k_3 + (1-\delta_3)k_2},$$
$$\hat{K}_{33} = \frac{k_1[k_2\delta_3(1-\delta_1)+k_3(1-\delta_3)]}{\delta_1 k_1 + (1-\delta_1)k_2}, \tag{18}$$
$$\hat{K}_{22}^{(1)} = k_1(1-\delta_1)\delta_3 + k_3(1-\delta_3).$$

The diagonal elements of the tensor of effective permeability of the blocks:

$$\hat{K}_{22}^{(2)} = k_2\delta_1\delta_3; \quad \hat{K}_{11}^{(2)} = \frac{k_2 k_3\delta_1\delta_3}{\delta_3 k_3 + (1-\delta_3)k_2}; \quad \hat{K}_{33}^{(2)} = \frac{k_1 k_2\delta_1\delta_2}{\delta_1 k_1 + (1-\delta_1)k_2}.$$

The effective porosity of the medium $\hat{m}$ is determined from Eq. (8):

$$\hat{m} = m_1(1-\delta_1)\delta_3 + m_2\delta_1\delta_3 + m_3(1-\delta_3). \tag{19}$$

In the case of a purely cracklike system, it was assumed in [4] that the flow in each crack is described by the Poiseuille formula. Thus, $k_i = \Delta_i^2/12$ and the formula for effective permeability $\hat{K}_{22}^{(1)}(k_1, 0)$ (18) coincides with the corresponding expression for permeability obtained in [4].

3. Effective Permeability of a Bed with Three Mutually Perpendicular Systems of Cracks. We will examine the case of equality of the degrees of openness and permeabilities of all cracks. Boundary-value problem (5-6) is formulated on a cube I^3. Proceeding as before, we construct the asymptote of its solution in small parameters of crack-opening with the accuracy $O((1-\delta)^2)$. Here, $N_i = N_i(\xi_i)$ is found by formulas similar to (16).

Calculations of the effective permeabilities show that the flows in both solid phases and in the system as a whole are isotropic:

$$\hat{K} = \frac{k_1[k_1(1-\delta^2)+k_2\delta^2]}{\delta k_1 + (1-\delta)k_2}, \quad \hat{K}^{(1)} = k_1(1-\delta^2)\left[1 + \frac{k_1(1-\delta^2)+k_2(2\delta^2-1)}{(1+\delta)\delta k_1 + (1-\delta^2)k_2}\right], \tag{20}$$
$$\hat{K}^{(2)} = k_2\left[1 - \frac{(1-\delta)[k_2+k_1(1+\delta)]}{\delta k_1 + (1-\delta)k_2}\right].$$

We use (8) to find the effective porosity $\hat{m} = m_1(1-\delta^3) + m_2\delta^3$.

It is evident from Eq. (20) that if the cracks in a cracked-porous system are impermeable, then the system is impermeable, $\hat{K}(0, k_2) = 0$. For a purely cracklike medium, with $k_2 = 0$, Eq. (20) for $\hat{K}(k_1, 0) = \hat{K}^{(1)}(k_1, 0)$ coincides with the formula obtained in [4].

It should be noted that the permeability of the system of cracks $\hat{K}^{(1)}$ depends not only on the permeability of the material of the cracks k_1, but also — as can be seen from (20) — on the intrinsic permeability of the blocks k_2. In particular, $\hat{K}^{(1)}$ does not coincide with the effective permeability of a purely cracklike system with expelled blocks. Thus, we cannot exclude the blocks from consideration when we write the law of motion in the crack system, as is done in the model of blocks in the form of distributed sources [3, 5]. The flow taking place through the blocks "suppresses" the filtration flow in the crack system. This mutual effect of the block and crack systems during filtration was observed in [8, 9] in descriptions of a cracked-porous medium in the form of a stochastic system.

The below formulas were obtained for the effective permeability and piezoconductivity of a medium in [5] using the model of blocks in the form of distributed sources in an examination of filtration in cracks and blocks:

$$\hat{K} = \hat{K}^{(1)}(k_1, 0), \quad \hat{\varkappa} = \frac{\hat{K}^{(1)}(k_1, 0)}{\beta\mu\,[3m_1(1-\delta) + m_2\delta^3/3]}\,.$$

It can be seen from the above formulas that at the third stage of filtration, the effective permeability of the cracked-porous medium coincides with the permeability of a purely cracklike system. This result is inconsistent with Eq. (20). In the formula for the piezoconductivity of the system $\hat{\kappa} = \hat{K}/\beta\mu\hat{m}$, the porosity of the blocks decreases by a factor of three. These differences are related to the fact that filtration in the blocks is not considered in the models in [3, 5].

Let us qualitatively analyze Eq. (20) for the effective permeability of a cracked-porous medium. The theorems in Parts 2 and 3 regarding the asymptotic representation of the solution of the cell problem were formulated with $1 - \delta \to 0$. Here, for any $k_1 > k_2$, we have:

$$\hat{K} = k_1\,\frac{1-\delta^2 + k_2\delta^2/k_1}{\delta + (1-\delta)\,k_2/k_1} \simeq 2(1-\delta)\,k_1 + \delta k_2. \tag{21}$$

Equation (21) can be interpreted as the permeability of a system consisting of a block and two cracks insulated from one another.

When the crack-opening parameter is small compared to the permeability ratio, we have $2(1 - \delta) \ll k_2\delta^2/k_1$. Equation (21) takes the form $\hat{K} = k_2\delta$. The permeability of the system is determined by the permeability of the blocks. When the permeability ratio is small compared to the crack-opening parameter, we have $2(1 - \delta) \gg k_2\delta^2/k_1$. Equation (21) takes the form $\hat{K} = 2(1 - \delta)k_1/\delta$. The permeability of the system is determined by the permeability of the cracks.

In the general case of differences in the properties of cracks growing in different directions, the solution asymptote of the cell problem has a form similar to (16). On the whole, the flows in systems of blocks and cracks are anisotropic. The expressions for the tensors of effective permeabilities $\hat{K}_{ij}$, $\hat{K}_{ij}^{(1)}$ and $\hat{K}_{ij}^{(2)}$, written in terms of k_i, Δ_i, and ℓ_i are omitted here due to their awkwardness.

4. Determination of the Parameters of the Bed from the Pressure-Drop Curve. We will examine the radial flow of a slightly compressible fluid from a bed dissected by three systems of mutually perpendicular cracks (Part 3) to a well operating with a constant yield q. At the first stage of filtration, the fluid moves only through cracks and the bed behaves as a homogeneous mass with the permeability of a purely cracklike system (with $k_2 = 0$) [1, 3, 5]. Here, the filtration process is described by the equation of the elastic regime (4) with piezoconductivity $\hat{\kappa}^{(1)} = \hat{K}^{(1)}/\mu\beta\hat{m}^{(1)}$, where $\hat{K}^{(1)}$ and $\hat{m}^{(1)}$ are determined by Eq. (20) with $k_2 = 0$, $m_2 = 0$.

At the third stage of filtration of the fluid, after the completion of mass exchange between the blocks and the cracks, the bed again behaves as a homogeneous mass [1, 3, 5]. The filtration process is described by Eq. (4) with the piezoconductivity $\hat{\kappa} = \hat{K}/\mu\beta\hat{m}$, where $\hat{K}$ and $\hat{m}$ were determined in Part 3.

The pressure drop Δp between the bed p_* and the bottom p_w is calculated from the formula [3]:

$$\Delta p = p_w - p_* = -\frac{q\mu B}{4\pi Kh} \ln \frac{4\varkappa t}{\gamma r_w^2}.$$

In Eq. (22), at the first stage of filtration, $K = \hat{K}^{(1)}(k_1, 0)$, $\kappa = \kappa^{(1)}$; at the third stage, $K = \hat{K}(k_1, k_2)$, $\kappa = \hat{\kappa}$. In logarithmic coordinates ($\ln t$, Δp), Eq. (22) is the equation of a straight line. We use the graphs of the straight lines for the first and third stages to determine the tangents of the slopes α_1 and α and the lengths of the segments intercepted by these lines on the y-axis b_1 and b. These constants are then used to express the effective permeability, crack permeability, and piezoconductivities

$$\hat{K}^{(1)}(k_1, 0) = -q B\mu/4\pi h\alpha_1, \quad \hat{K}(k_1, k_2) = -q B\mu/4\pi h\alpha,$$
$$\hat{\varkappa}^{(1)} = \frac{\gamma r_w^2}{4} \exp\left(\frac{b_1}{\alpha_1}\right), \quad \hat{\varkappa} = \frac{\gamma r_w^2}{4} \exp\left(\frac{b}{\alpha}\right).$$

All four parameters are functions of the following five characteristics of the reservoir: the permeabilities of the matrix k_2 and crack k_1, their porosities m_2 and m_1, the ratio of the opening of the crack to the length of the block Δ/l. By assigning any of these characteristics (such as k_2 or m_2, using the results of core studies), we can find the other four.

In the case of the flow of oil toward a series of wells from a layered-inhomogeneous bed with periodic alternation of thin, high-permeability and thick, low-permeability layers, the values of $\hat{K}^{(1)}$, $\hat{K}$, $\hat{m}^{(1)}$, $\hat{m}$ determined from the above formulas on the basis of a well analysis are expressed through the parameters of the internal structure k_1, k_2, m_1, m_2, Δ/l using Eqs. (8) and (15).

5. Determination of the Parameters of the Bed from Pressure-Recovery Curves. We will examine the process of radial filtration in a bed after the shutdown of a well. The pressure drop Δp between the bed p_* and the bottom of the well p_w at the moment $t_p + \Delta t$ after shutdown of the well is calculated from the formula [3]:

$$p_w(t_p + \Delta t) - p_* = -\frac{q\mu B}{4\pi Kh}\left[\ln \frac{4\hat{\varkappa}(t_p + \Delta t)}{\gamma r_w^2} - \frac{\hat{K}}{K} \ln \frac{4\varkappa\Delta t}{\gamma r_w^2}\right]. \tag{23}$$

Similar to Part 4, we assume that at small Δt, in Eq. (23) $K = \hat{K}^{(1)}(k_1, 0)$, $\kappa = \hat{\kappa}^{(1)}$. Here (23) takes the form

$$p_w(t_p + \Delta t) - p_* = -\frac{q\mu B}{4\pi h\hat{K}}\left[\ln(t_p + \Delta t) - \frac{\hat{K}}{\hat{K}^{(1)}} \ln \Delta t + \ln \frac{4\hat{\varkappa}}{\gamma r_w^2} - \frac{\hat{K}}{\hat{K}^{(1)}} \ln \frac{4\hat{\varkappa}^{(1)}}{\gamma r_w^2}\right]. \tag{24}$$

At large values of Δt, in (23) we have $K = \hat{K}(k_1, k_2)$. After insertion of the effective permeability $\hat{K}$ into Eq. (23), we have

$$p_w(t_p + \Delta t) - p_* = -\frac{q\mu B}{4\pi\hat{K}h} \ln \frac{t_p + \Delta t}{\Delta t}. \tag{25}$$

By analyzing measurements of bottom-hole pressure at large Δt using Eq. (25) in logarithmic coordinates ($\ln[(t_p + \Delta t)/\Delta t]$, $p_w - p_*$), we can determine the value of $\hat{K}$ from the slope tangent of the straight line λ:

$$\lambda = q\mu B/4\pi\hat{K}h. \tag{26}$$

We use Eq. (24) for two small values of time Δt and $\Delta t'$:

$$p_w(t_p+\Delta t)-p_w(t_p+\Delta t') = -\frac{q\mu B}{4\pi\hat{K}h}\left[\ln\frac{t_p+\Delta t}{t_p+\Delta t'}-\frac{\hat{K}}{\hat{K}^{(1)}}\ln\frac{\Delta t}{\Delta t'}\right]. \tag{27}$$

Equation (27) makes it possible to find the value of $\hat{K}^{(1)}$. Then Eq. (24) relates the piezoconductivity of the system as a whole $\hat{\kappa}$ to the piezoconductivity of the system of cracks $\hat{\kappa}^{(1)}$ (or the porosities $\hat{m}$ and $\hat{m}^{(1)}$).

This relation is simplified in the case of an impermeable matrix. In this case, $\hat{K} = \hat{K}^{(1)}$, and Eqs. (24) and (25) become the equations of parallel straight lines on the plane $[\ln[(t_p + \Delta t)/\Delta t], p_w - p_*]$.

By measuring the distance D along the vertical between these lines, we obtain the relation

$$\frac{3m_1(1-\delta)}{3m_1(1-\delta)+m_2} = \exp\left(-\frac{D}{\lambda}\right). \tag{28}$$

The left side of (28) corresponds to the fraction of fluid located in the cracks.

Example. The study [5] presented results of an analysis of a well from the pressure-recovery curve. The yield $q = 4.7\cdot10^{-3}$ m^3/sec, $B = 2.3$, $h = 100$ m, $\beta = 1.2\cdot10^{-9}$ Pa^{-1}, $\mu = 10^{-3}$ Pa$\cdot$sec, $m_2 = 0.21$, and $m_1 = 0.5$. Straight lines (24) and (25) are parallel, with the slope tangent $\lambda = \lambda_1 = 0.945\cdot10^5$ Pa and the distance $D = 7.29\cdot10^5$ Pa. We find from Eq. (28) that the cracks contain 0.0446% of the oil. We then use this formula to determine the degree of opening of the cracks $\Delta/l = 1 - \delta = 6.2\cdot10^{-5}$. We find from (26) that $\hat{K} = 0.09238\cdot10^{-12}$ m^2, from which $\kappa = 0.37$ m^2/sec. We use (21) to determine the intrinsic permeability of the cracks $k_1 = 0.745\cdot10^{-9}$ m^2.

NOTATION

t, time; p, pressure; μ, viscosity; p_*, initial formation pressure; L, length of the bed; x_i, coordinate of the three-dimensional space; $x = (x_1, x_2, x_3)$, vector; **k**, permeability of the porous medium; m, porosity of the medium; $n = (n_1, n_2, n_3)$, normal vector to the surface of discontinuity of the functions k and m; [f], difference between two values of the function f on both sides of the surface; q, yield of well; k_1, permeability of crack; k_2, permeability of matrix; m_1, porosity of cracks; m_2, porosity of matrix; Δ, characteristic crack-opening; l, characteristic dimension of block, $\delta = l/(l + \Delta)$; $p_w(t)$, bottom-hole pressure at the moment t; r_w, radius of well; $I^3 = I \times I \times I$, unit cube; Γ_1, volume of cell corresponding to the cracks; Γ_2, volume of cell corresponding to a block; β, compression modulus of the fluid; B, coefficient of volume shrinkage of the oil due to its degassing; Δt, time of operation of well after shutdown; $\gamma = 1.781076$, Euler constant. Indices: r = 1, crack; r = 2, matrix.

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